Large views of small phenomena: decompositions, localizations, and representation type (LAVIE)

REPRESENTATION THEORY OF ALGEBRAS is an extremely lively and visible research area with close ties to many other fields, such as algebraic geometry, modular representation theory, Lie theory, mathematical physics, or topological data analysis. An associative algebra A is studied via its representations, that is, via the category of A-modules and its derived category. Reduction techniques play an important role in this context. A strategy widely employed in algebra, geometry, and topology is the decomposition of a category into smaller parts that are still big enough to reconstruct the whole category. This leads to the notion of a torsion pair and to the fundamental tool of localization. A further reduction technique consists in decomposing modules into indecomposable summands; the indecomposable A-modules can often be classified and yield information on the whole module category. The representation type of A is a measure for the complexity of the module category and decides whether such classification is possible or not.

OUR VIEWPOINT. Problems related to the representation type of an algebra concern small, that is, finite dimensional modules. But they are controlled by large, that is, possibly infinite dimensional modules. We propose a novel approach that takes into account the interplay between small and large objects and is based on recent advances on infinite dimensional modules achieved e.g. in silting theory and localization theory.

METHODS. The project builds on a variety of different methods of inquiry and requires expertise from different areas of mathematics, including representation theory, model theory, commutative algebra, algebraic geometry, and geometric invariant theory. It will be carried out by an interdisciplinary team combining the following independent, but closely interrelated approaches.

1. **The lattice torsA of torsion classes** in the category of finite dimensional modules over a finite dimensional algebra A is currently receiving a lot of attention due to its connection with silting theory and mutation. Silting theory is a young and dynamic branch of representation theory with many unexpected connections to other areas. One of its highlights is the interplay with cluster algebras: clusters are interpreted as silting objects and cluster mutation as an operation that exchanges summands in silting objects. Silting mutation is reflected in a part of the lattice torsA. We propose a new approach to mutation, based on the dual concept of a cosilting object, which captures the whole lattice torsA. This requires to drop the restriction to small modules and to work with large cosilting complexes. Since the latter are pure-injective, our results will also have an interpretation in terms of the Ziegler spectrum of A, a topological space originating from model theory. We will develop an abstract framework at the level of triangulated categories and address the question whether all cosilting objects are pure- injective.

2. **The ring epimorphisms with fixed domain A** form another lattice encoding valuable information on an algebra A. We will investigate the interaction of this lattice with torsA. Ring epimorphisms with nice homological properties, notably universal localizations, are closely related with (co)silting objects and with certain decompositions of derived categories. We will study these connections, building on the well-understood case of a commutative noetherian ring. We will also explore finiteness conditions on universal localizations and their impact on representation type.

3. **The concept of stability** originates in algebraic geometry and appears in many different contexts, ranging from geometric invariant theory to quantum field theory, and mirror symmetry. Bridgeland showed that stability conditions over an algebra A are encoded in a wall and chamber structure. These data can be described in terms of silting theory, interpreting mutation as wall crossing. But not all walls are captured by this approach. We will investigate these phenomena within the broader framework of large (co)silting theory. We also want to uncover the links with universal localization emerging from King’s work. This involves the concept of a rank function which is currently object of renewed interest.

4. **Case studies** over distinguished families of algebras (canonical algebras, Brauer graph algebras, triangular matrix algebras, hereditary orders) will complement our perspective. Here the focus will lie on classification results for pure-injective and/or (co)silting objects.